NAG Toolbox for MATLAB

g01ha

1 Purpose

g01ha returns the lower tail probability for the bivariate Normal distribution, via the function name.

2 Syntax

```
[result, ifail] = g01ha(x, y, rho)
```

3 Description

For the two random variables (X, Y) following a bivariate Normal distribution with

$$E[X] = 0, \qquad E[Y] = 0, \qquad E\left[X^2\right] = 1, \qquad E\left[Y^2\right] = 1 \qquad \text{and} \qquad E[XY] = \rho,$$

the lower tail probability is defined by:

$$P(X \leq x, Y \leq y : \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^{y} \int_{-\infty}^{x} \exp\left(-\frac{\left(X^2 - 2\rho XY + Y^2\right)}{2(1-\rho^2)}\right) dX\,dY.$$

For a more detailed description of the bivariate Normal distribution and its properties see Abramowitz and Stegun 1972 and Kendall and Stuart 1969 The method used is described by Divgi 1979.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Divgi D R 1979 Calculation of univariate and bivariate normal probability functions *Ann. Statist.* **7** 903–910

Kendall M G and Stuart A 1969 *The Advanced Theory of Statistics (Volume 1)* (3rd Edition) Griffin

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

x, the first argument for which the bivariate Normal distribution function is to be evaluated.

2: **y – double scalar**

y, the second argument for which the bivariate Normal distribution function is to be evaluated.

3: **rho – double scalar**

ρ, the correlation coefficient.

Constraint: $-1.0 \leq$ **rho** ≤ 1.0.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **rho** < -1.0,
or **rho** > 1.0.

If on exit **ifail** $= 1$ then g01ha returns zero.

7 Accuracy

Accuracy is discussed in Divgi 1979. A higher order polynomial approximation to Mills ratio is used in g01ha, (15 terms) than is given in Divgi 1979. This will give higher absolute accuracy of about 10 digits on machines of sufficiently high precision.

8 Further Comments

The probabilities for the univariate Normal distribution can be computed using s15ab and s15ac.

9 Example

```
x = 1.7;
y = 23.1;
rho = 0;
[result, ifail] = g01ha(x, y, rho)
```

```
result =
    0.9554
ifail =
           0
```